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RANDOM-NUMBER GUESSING AND THE FIRST DIGIT PHENOMENON[1]

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Summary.—To what extent do individuals "absorb" the empirical regularities of their environment and reflect them in behavior? A widely-accepted empirical observation called the First Digit Phenomenon or Benford's Law says that in collections of miscellaneous tables of data (such as physical constants, almanacs, newspaper articles, etc.), the first significant digit is much more likely to be a low number than a high number. In this study, an analysis of the frequencies of the first and second digits of "random" six-digit numbers guessed by people suggests that people's responses share some of the properties of Benford's Law: first digit 1 occurs much more frequently than expected; first digit 8 or 9 occurs much less frequently; and the second digits are much more uniformly distributed than the first.

It seems to be widely accepted that people cannot behave truly randomly, even in situations such as game-playing where success may depend on an ability to perform in an unpredictable way. If people are asked to generate random numbers, their responses differ significantly from truly random sequences; Tune (1964) has a good review of the literature.

For example, an experiment of T. Varga [see Revesz (1978) or a related experiment by Bakan (1960)] is this. Half a class of students is asked to flip a coin 200 times and record the results; the other half is asked to fake a sequence of 200 tosses. By later declaring "fake" any sequence which fails to have a run of at least length six, Varga found he could distinguish between the true and the faked sequences with very high accuracy. Of course, once this rule is known the fakers can beat it, and in general it seems possible to be able to learn to generate more random sequences (cf. Neuringer, 1986).

As a second example, a statistical analysis of the winners of the Massachusetts Numbers Game by Chernoff (1981) yielded a collection of 33 four-digit numbers which were sufficiently unlikely to be picked by players as to make them potentially favorable bets. In that game, first the players bet on a four-digit number of their own choice, next a single four-digit number is drawn by a judge (or generated randomly), and then all players with the winning number share the pot equally. In such a situation it is advantageous to be able

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to identify numbers which few people choose, since all numbers are equally likely to be winners and the expected payoff for an unpopular number is then higher than that for a number which many people have chosen. (Of course, "unpopular" numbers become *popular* numbers as soon as they are learned.)

Perhaps when people think they are generating a random number they are often basing it on numbers from their own experience, and if the numbers in their experience are not uniformly distributed (or "truly random"), this should be reflected in the responses. A widely quoted empirical observation, apparently first published by the physicist Benford (1938), is that randomly occurring tables of data tend to have entries that begin with low numbers: in particular, the first significant digit is 1 almost three times as much as one would expect and is 9 less than half as much as one would expect.

The purpose of this research was to explore the possible connection between this First Digit Phenomenon and the responses of people trying to generate random numbers. If people are influenced by experience with numbers roughly obeying Benford's observations, one would expect their responses to have many more numbers beginning with 1 or 2 than with 8 or 9; this was the case, although not nearly as extremely as in Benford's data.

First Digit Phenomenon

Benford observed that tables of logarithms in libraries tend to be progressively dirtier near the beginning and wondered why students of science and engineering (say) have more occasion to calculate with numbers beginning with 1 or 2 than with 8 or 9. He studied 20 tables of numbers including molecular weights of chemicals, surface areas of rivers, street addresses of famous people, baseball statistics, arithmetical sequences, and newspaper items and found that the leading significant (nonzero) digit in his data was 1 with frequency .306, as opposed to the uniform frequency 1/9 one would expect for a "truly random" distribution of the first significant digits.

In general, Benford found empirically that the proportion of entries beginning with first (nonzero) digit d is well approximated by

$$\log_{10}(d+1) - \log_{10}(d),\ d = 1,2,\ldots,9. \qquad [1]$$

There have subsequently been many theoretical models offered to explain [1], including those by Cohen (1976), Diaconis (1977), Flehinger (1966), and Raimi (1969, 1976); Raimi (1976) has a good survey of the literature on this problem.

Similarly, Benford found that the frequencies of digits k places from the left are also nonuniform, although they do tend to be more uniformly distributed as k increases. The proportion of entries with second significant digit d is approximately

$$\log_{10} \prod_{j=1}^{9} (10j + d + 1) - \log_{10} \prod_{j=1}^{9} (10j + d), d = 0,1,...,9. \quad [2]$$

For example, the second significant digit is 1 with probability $\log_{10}(12 \cdot 22 \cdots 92) - \log_{10}(11 \cdot 21 \cdots 91) \cong .114$; Table 1 below includes the relative frequency of the first and second digits according to the laws [1] and [2].

THE EXPERIMENT

Method

A large number (742) of undergraduate calculus students were given slips of paper and asked to write down a six-digit random number "out of their heads," i.e., no calculators, coin-flipping, etc. The frequencies of the first and second significant digits in their responses were calculated and are summarized in the following table, along with the theoretical frequencies of the first and second digit laws [1] and [2] (which are referred to as Benford frequencies in the table), and the "truly random" or uniform distributions. (Two responses of all zeroes were not included in the table.)

The choice of number of digits requested (six) was made at A. Tversky's suggestion. Many natural variations of the experiment are possible, among them: requesting a different, or unspecified number of digits; allowing more time, say several days, for responses; including rewards of some type, such as a lottery jackpot as in the numbers game studied by Chernoff or a reward structure of Neuringer (1986); using subjects from different classes of mathematical sophistication [Chapanis (1953) found that more mathematically sophisticated subjects responded with more nearly uniform (random) sequences]; and allowing the subjects to call out, use computer keyboard, or otherwise register their responses.

Results

Analysis of the data in Table 1 was made using the chi-squared and Kolmogarov-Smirnov goodness-of-fit tests. Other tests such as measure of information content or autocorrelation are also possible; the reader is referred to

TABLE 1
FREQUENCIES OF FIRST AND SECOND SIGNIFICANCE DIGITS

First Digit	Observed	Benford (1)	Uniform	Second Digit	Observed	Benford (2)	Uniform
1	.147	.301	.111	0	.058	.120	.100
2	.100	.176	.111	1	.106	.114	.100
3	.104	.125	.111	2	.117	.109	.100
4	.133	.097	.111	3	.109	.104	.100
5	.097	.079	.111	4	.105	.100	.100
6	.157	.067	.111	5	.100	.097	.100
7	.120	.058	.111	6	.112	.093	.100
8	.084	.051	.111	7	.128	.090	.100
9	.058	.046	.111	8	.073	.088	.100
				9	.092	.085	.100

Neuringer (1986) for references. As one would expect by looking at Table 1 (left), the null hypothesis that the response frequencies obey Benford's Law is rejected at the .05 significance level by both the chi-squared (8 *df*) test and by the Kolmogarov-Smirnov test. Table 1 (right) shows better agreement; the chi-squared (9 *df*) test permits acceptance of the null hypothesis that the true distribution is Benford at the .05 significance level whereas the Kolmogarov-Smirnov test requires rejection; both tests permit acceptance of the null hypothesis that the true distribution is uniform at the .05 level.

Discussion

Although not conforming precisely to the predictions of the First Digit Law, the results of the experiment indicate that the distributions of random numbers guessed by people share the following properties with the Benford distributions: (i) the frequency of numbers with first significant digit 1 is much higher than expected; (ii) the frequency of numbers with first significant digit 8 or 9 is much lower than expected; and (iii) the distribution of the second digits is much more nearly uniform than the distribution of the first digits.

These conclusions are consistent with Chernoff's (1981) findings that generally high numbers are less likely to be chosen in numbers games. Of the 33 numbers in his "first system" (numbers with predicted normalized payoffs exceeding 1.0), 16 had first significant digit 8 or 9, and only one has first significant digit 1 or 2. (Recall that Chernoff tried to identify numbers which were *unlikely*, so a high proportion of large numbers in his list corresponds to a low frequency of occurrence of high numbers.)

These conclusions are also undoubtedly related to other known common response phenomena such as the tendency, in interest surveys, to select the first choice early in the list. Accordingly, these results seem quite germane for psychometricians involved in the design of tests.

Several other questions are raised by this experiment. For example, Table 1 (left) suggests that numbers with leading significant digit 5 are much less likely to occur than those with 4 or 6, at least by calculus students. Is this true in general, and if so, why? Conclusion (iii) above suggests that people do not generate random numbers digit-by-digit but rather according to some other rule. Is this true in general? The data in Table 1 also suggests a tendency towards bimodality, with local minima at 5 or 6. Does this perhaps reflect two separate subpopulations with one choosing numbers early in the series and one choosing from the high end?

REFERENCES

BAKAN, P. (1960) Response-tendencies in attempts to generate binary series. *Amer. J. Psychol.*, 73, 127-131.

BENFORD, F. (1938) The law of anomalous numbers. *Proc. Amer. Philos. Soc.*, 78, 551-572.

CHAPANIS, A. (1953) Random-number guessing behavior. *Amer. Psychologist*, 8, 332.

CHERNOFF, H. (1981) How to beat the Massachusetts Numbers Game. *Math. Intel.*, 3, 166-172.

COHEN, D. (1976) An explanation of the first digit phenomenon. *J. Combinatorial Th.*, (A) 20, 367-370.

DIACONIS, P. (1977) The distribution of leading digits and uniform distribution mod 1. *Ann. Probab.*, 5, 72-81.

FLEHINGER, B. (1966) On the probability that a random integer has initial digit A. *Amer. Math. Mo.*, 73, 1056-1061.

NEURINGER, A. (1986) Can people behave "randomly"?: the role of feedback. *J. exp. Psychol.* (Gen.), 115, 62-75.

RAIMI, R. (1969) The peculiar distribution of first significant digits. *Sci. Amer.*, 221(6), 109-120.

RAIMI, R. (1976) On the distribution of first significant digits. *Amer. Math. Mo.*, 76, 342-348.

REVESZ, P. (1978) Strong theorems on coin-tossing. *Proc. Internat. Congr. of Mathematicians, Helsinki*, 749-754.

TUNE, G. (1964) Response preferences: a review of some relevant literature. *Psychol. Bull.*, 61, 286-302.

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